

Characterization of 1D Linear Piecewise-Smooth Discontinuous Map

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Abstract—One-dimensional (1D) linear piecewise-smooth discontinuous (LPSD) map with one jump discontinuity are known to exhibit a rich and complex dynamics ranging from fixed point to periodic orbits and even chaotic orbits. In this paper, we have given a complete characterization of 1D LPSD map. Further, with the help of cobweb plots, the theoretical analysis is validated. We have considered three different quadrants to illustrate behaviour of these maps. Moreover, the summary of analysis of all the results is presented in a table.

Index Terms—Piecewise-smooth maps, periodic orbit, chaotic orbit.

I. INTRODUCTION

Piecewise-smooth systems form a typical class of nonlinear systems. These represent an vital and effective modeling tool to approximate nonlinear systems. In practice, different physical systems, economic models and various other fields comprises of discontinuity or sudden change. Examples of such phenomenon occurs in economical models of financial markets, neuron model in biology, impact oscillators in mechanical systems and switching circuits like DC-DC converters in electrical networks [1].

In power electronic circuits instability, sub-harmonic phenomenon and chaos are predicted and observed [2]. A linear increasing map in left side and a linear decreasing map in right side are introduced for canonical 1D discontinuous map and the properties of different regions were studied [3]. In [4] the analytic approach of the existence of stable periodic orbits and novel approach to find the admissible patterns was proposed.

In this paper we have carried out a complete characterization of 1D LPSD map. This paper is organized in the following way. In Section II we introduce the basics of analytical method used in the paper and few mathematical preliminaries. In Section III we analyse all regions of all quadrants of ab -plane. Section IV presents the summary followed by conclusions in Section V.

II. THE MATHEMATICAL SET-UP

The 1D LPSD map is defined as [5]

$$x_{n+1} = f(x_n, a, b, \mu, \ell) = \begin{cases} ax_n + \mu & \text{for } x_n \leq 0 \\ bx_n + \mu + \ell & \text{for } x_n > 0 \end{cases} \quad (1)$$

Here, a , b and μ are the slopes of the map and ℓ is the height of jump discontinuity. The real line is divided into two halves or

sets. These sets are denoted by two symbols namely; the closed left set as $\mathcal{L} := (-\infty, 0]$ and open right set as $\mathcal{R} := (0, \infty)$. Depending on values of parameters a and b , the map (1) can be analyzed with a, b in four quadrants of ab -plane as shown in Fig. 1.

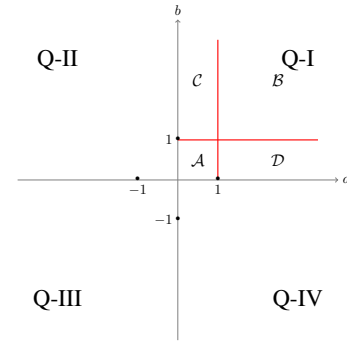


Fig. 1: The parameter ab -plane with four quadrants.

Each quadrant is further divided in four regions named as A, B, C and D (see Fig. 1). The complete characterization of orbits for Quadrant-I is carried out by in References [4, 6]. For Quadrant-II and Quadrant-III, the the complete characterization of orbits is yet to be carried out. In this paper we have mathematically analyzed the various possible scenarios for Quadrant-II and III. Further, we have validated our results with numerical simulations and cobweb plots. This analysis would serve as the basic building block for carrying out complete characterization of orbits in Quadrant-II and III.

The basic definitions of periodic orbit, pattern, range of existence are explained in Ref. [4]. We will use these terms and would start analyzing the Quadrants.

III. ANALYSIS

Recall the map equation for 1D LPSD map given by Equation (1). We now analyze Quadrant-II and III.

A. Quadrant-II

In this quadrant a is negative and b is positive. Here we analyse the regions (A', B', C', D'). These are similar to 4 regions analysed for Quadrant-I [4, 6].